SELECTED SOLUTIONS

CHAPTER 9

Exercise Set 9.1.1

1. Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

2. Symmetry with respect to origin because $f(-x) = -f(x)$

3. Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

4. Symmetry with respect to y-axis because $f(-x) = f(x)$

5. The values of $1/x$ get closer and closer to 0 as x gets larger.

Exercise Set 9.1.2

1. $x \neq -2$ or $(-\infty, -2) \cup (-2, \infty)$

2. All real numbers or $(-\infty, \infty)$

3. $x \neq \pm 2$ or $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

4. $x \neq -5, -2, ½$ or $(-\infty, -5) \cup (-5, -2) \cup (-2, ½) \cup (½, \infty)$

5. $x \neq -2$ or $(-\infty, -2) \cup (-2, \infty)$

Exercise Set 9.1.3

1. [ ] (a set with no elements)

2. 'EmptySet'

3. **solveset** returns a set. To convert to a sorted list, insert the following line above the **print** statement:

```
nonzeros = sorted(list(nonzeros))
```

4. Replace **print**(nonzeros) with the following:

```
if len(nonzeros) == 0:
```

```
  print("none")
else:
  print(nonzeros)
```

Exercise Set 9.1.4

1. `domain += “,”+str(nz)+”)U(“+str(nz)`

2. Add the following three lines:

```
factored = factor(poly)

print("Factorization of denominator:")
print(factored)
```

3. Nonzeros:

[-2, -1/4, 3]

Domain:

(-∞ ,-2)U(-2,-1/4)U(-1/4,3)U(3, ∞)

Factorization of denominator:

(x - 3)*(x + 2)*(4*x + 1)

2. Add the following three lines:

```
factored = factor(poly)

print("Factorization of denominator:")
print(factored)
```

Exercise Set 9.1.5

1. x = 11 VA

2. x = 10 VA

3. x = 2 RD, x = -5/2 VA

4. x = -4 RD x = 5 VA

5. x = 2 VA

6. x = -1/3 RD x = 5 VA

7. any function that is a constant multiple of f(x) = $\frac{(x-5)}{(x-5)(x-1)(x+7)}$

Exercise Set 9.1.6

1.

```
>>> rational_fcn='(2*x**3+3*x**2-3*x-2)/(x**4+2*x**3-21*x**2-22*x+40)'
>>>num,den = numer(rational_fcn),denom(rational_fcn)
>>>num
2*x**3 + 3*x**2 - 3*x - 2
>>>den
x**4 + 2*x**3 - 21*x**2 - 22*x + 40
```

i.e. the numerator is assigned to string variable **num** and the denominator is assigned to string variable **den**.

2. (x - 4)*(x - 1)*(x + 2)*(x + 5)

3. (2*x + 1)/(x**2 + x - 20), or $\frac{2x+1}{x^2+x-20}$.

4. (x-4)(x+5)

5. They are not the same because the cancelled version of the denominator (in #4) has fewer factors than the uncancelled denominator (in #2).

6. The vertical asymptotes are at x = 4 and x = -5. The removable discontinuities are at x = 1 and x = -2.

7. f(1) = -1/6 and f(-2) = 1/6.

Exercise Set 9.1.7

1. Replaced yellow highlighted code to print and classify the list of original nonzeros.

```
if nz in reduced_nonzeros:
    print(nz,", vertical asymptote")
else:
    print(nz,", removable discontinuity")
```

2. Replace code in #1 above with the following:

```
if nz in reduced_nonzeros:
    print("vertical asymptote at x=", nz)
else:
    y = reduced_form.subs({x:nz})
    print("removable discontinuity at (",nz,",",y, ")")
```

3. Replace

```
rational_fcn='(2*x**3+3*x**2-3*x-2)/(x**4+2*x**3-21*x**2-22*x+40)'
```

with:

```
rational_fcn = input('Rational Function? ')
```

4. Rational Function? (2*x**3+3*x**2-3*x-2)/(x**4+2*x**3-21*x**2-22*x+40)
Reduced form:
(2*x + 1)/(x**2 + x - 20)
Nonzeros:
vertical asymptote at x= -5
removable discontinuity at (-2 , 1/6)
removable discontinuity at (1 , -1/6)
vertical asymptote at x= 4

Exercise Set 9.2.1

1. x-int: (0,0); y-int: (0,0)

2. x-int: none; y-int: (0,3)

3. x-int: (-4,0), (4,0); y-int: (0, 16/81)

4. x-int: (2,0), (5,0); y-int: none

5. x-int: (7,0); y-int: (0, 7/2)

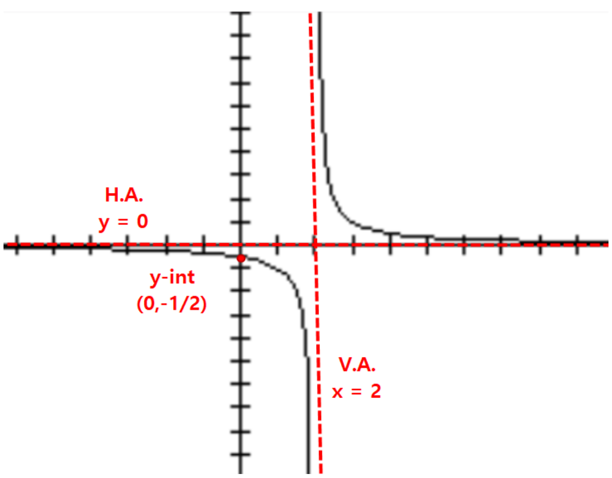
6. Write a program that simplifies a rational function, then identifies its x- and y-intercepts.

```
from sympy import *
x = Symbol('x')
r = sympify(input('rational expression? '))
```

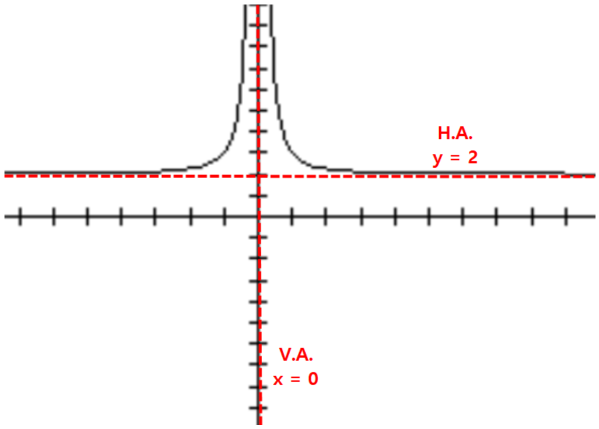
```
### simplify the rational expression
r2 = simplify(r)
print('Simplified function is', r2)


### find the x-intercept(s) by setting numerator = 0
num = numer(r2)
x_int = solve(num)
print('x-intercepts:')
k = 0
for a in x_int:
  if a.is_real:
    print(a)
    k +=1
if k==0:
  print('none')

### y-intercept is f(0)
y_int = r.subs({x:0})
print('y-intercept:')
print(y_int)
```

Exercise Set 9.2.2

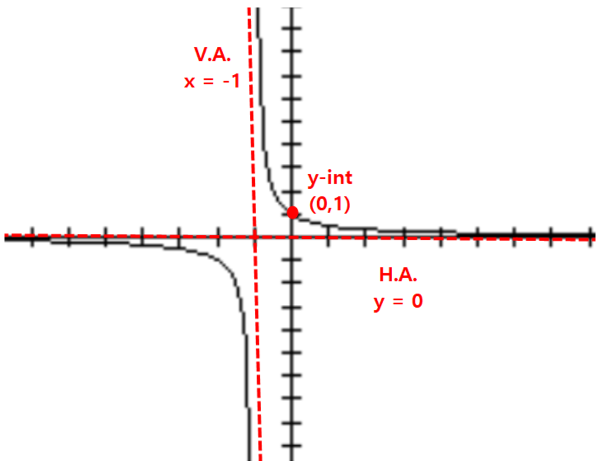
1. $f(x) = \frac{1}{x}$ shifted 2 units right. No x-int.



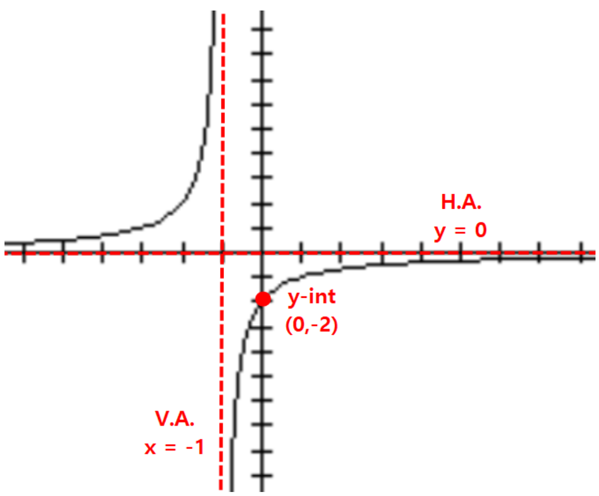
2. $f(x) = \frac{1}{x^2}$ shifted 2 units up. No x- or y-intercepts.



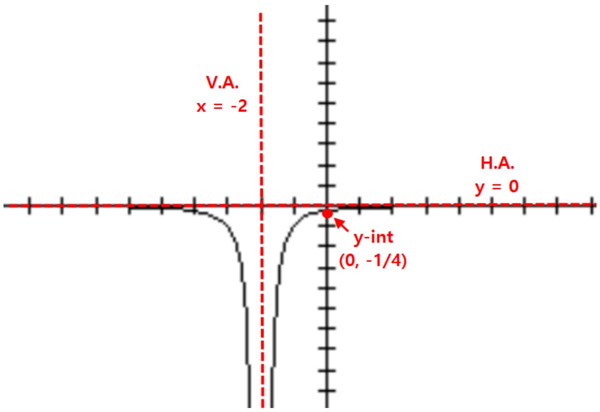
3. Simplifies to $f(x) = \frac{1}{x+1}$ which is $f(x) = \frac{1}{x}$ shifted 1 unit left. No x-int.



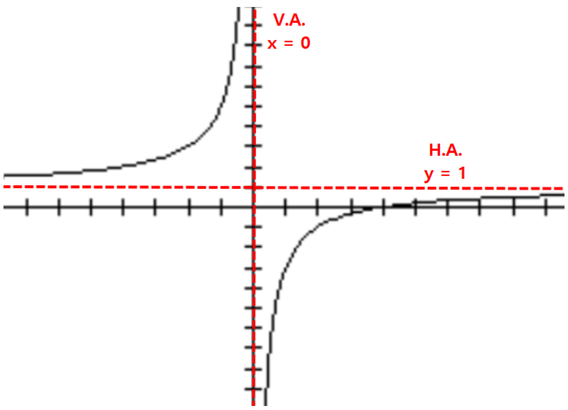
4. $f(x) = \frac{1}{x}$ shifted 1 unit left, stretched by a factor of 2, and reflected through the x-axis. No x-int.



5. Simplifies to $f(x) = -\frac{1}{(x+2)^2}$ which is $f(x) = \frac{1}{x^2}$ shifted 2 units left and reflected through the x-axis. No x-int.



6. Simplifies to $f(x) = 1 - \frac{4}{x}$ which is $f(x) = \frac{1}{x}$ stretched by a factor of 4, reflected through the x-axis, and shifted up 1 unit. No intercepts.



4. $y = -\frac{1}{x+2}$

5. $y = \frac{1}{x^2} - 3$

Exercise Set 9.2.3

1. H.A.: y = 1; V.A.: x = -3, x = 2

2. H.A.: y = 0; V.A.: x = -5, x = 5

3. H.A.: y = 0; V.A. x = -3, x = 3

4. H.A.: none; V.A. x = -3

5. Add the following code to the bottom of the program:

```
# calculate and draw horizontal asymptotes
nump = Poly(num,x)
num_coeffs = nump.all_coeffs()
num_degree = nump.degree()
denp = Poly(den,x)
den_coeffs = denp.all_coeffs()
den_degree = denp.degree()
if num_degree<den_degree:
   ha = 0
elif num_degree==den_degree:
   ha= num_coeffs[0]/den_coeffs[0]
else:
   ha='none'

if ha!="none":
   plt.plot([-20,20],[ha,ha],color="green")
   ha_eqn='y='+str(ha)
   plt.annotate(ha_eqn,xy=(15,ha))
```

Exercise Set 9.2.4

1. V.A.: x = -4, x = 2; H.A.: y = 0; O.A.: none; Holes: none; x-int: (0,0); y-int: (0,0)

2. V.A.: x = 2; H.A.: none; O.A.: y = -x – 5; Holes: none; x-int: (-4,0), (1,0); y-int: (0, -2)

3. V.A.: x = -1, x = 1; H.A.: y = 1; O.A.: none; Holes: none; x-int: (-2,0), (3,0); y-int: (0,6)

4. V.A.: x = -5, x = 1; H.A.: y = 0; O.A.: none; Holes: none; x-int: (-1/2, 0); y-int: (0, -2/5)

5. V.A.: x = -1.5439; H.A.: y=0; O.A.: none; Holes: none; x-int: (0,0), (-2/3, 0); y-int: (0,0)

6. V.A.: x = -3.3028, x = 0.30278; H.A.: none; O.A.: y = 5x – 15; Holes: none; x-int: (1.1696,0); y-int: (0, 8)

7. It is not possible to have both a horizontal and oblique asymptote. A horizontal asymptote only occurs when the degree of the numerator is the same as, or less than, the degree of the denominator. An oblique asymptote only occurs when the degree of the numerator is exactly one more than the degree of the denominator.

8. Insert the following code at the end of the program:

```
if num_degree==den_degree+1:  # oblique asymptote
   quo,rem = div(num,den)
   # draw oblique asymptote
   x_list=[-20,20]
```

```
y_list=[quo.subs({x:x_value}) for x_value in x_list]
plt.plot(x_list,y_list,color='purple')
# label the graph with the equation of the oblique asymptote
y = quo.subs({x:-20})
oblique='y='+str(quo)
plt.annotate(oblique,xy=(-20,y))
```

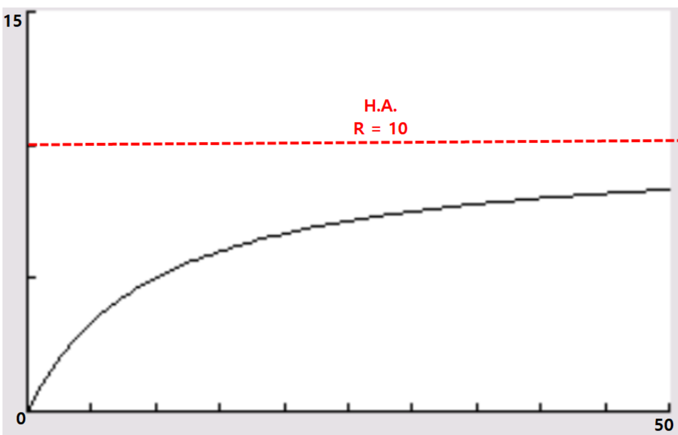
Exercise Set 9.3.1

1. A. 9.8208 m/sec^2

   B. g = 0

   C. About 9.818 m/sec^2

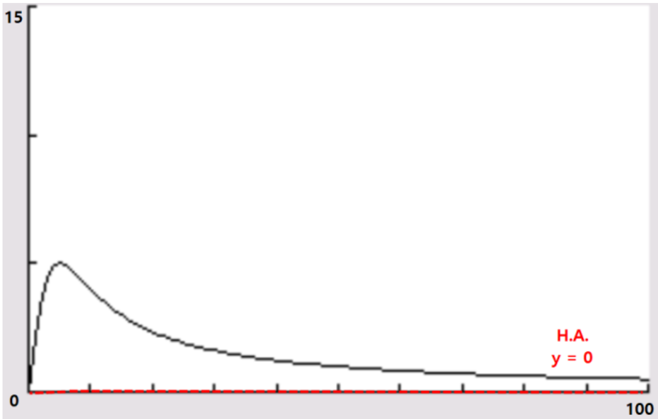
2. A. Graph



   B. R_{tot} = 10. As the resistance R_2 increases without bound, the total resistance approaches 10 ohms, the resistance R_1.

3. A. C = 0. The concentration diminishes.

   B. Graph

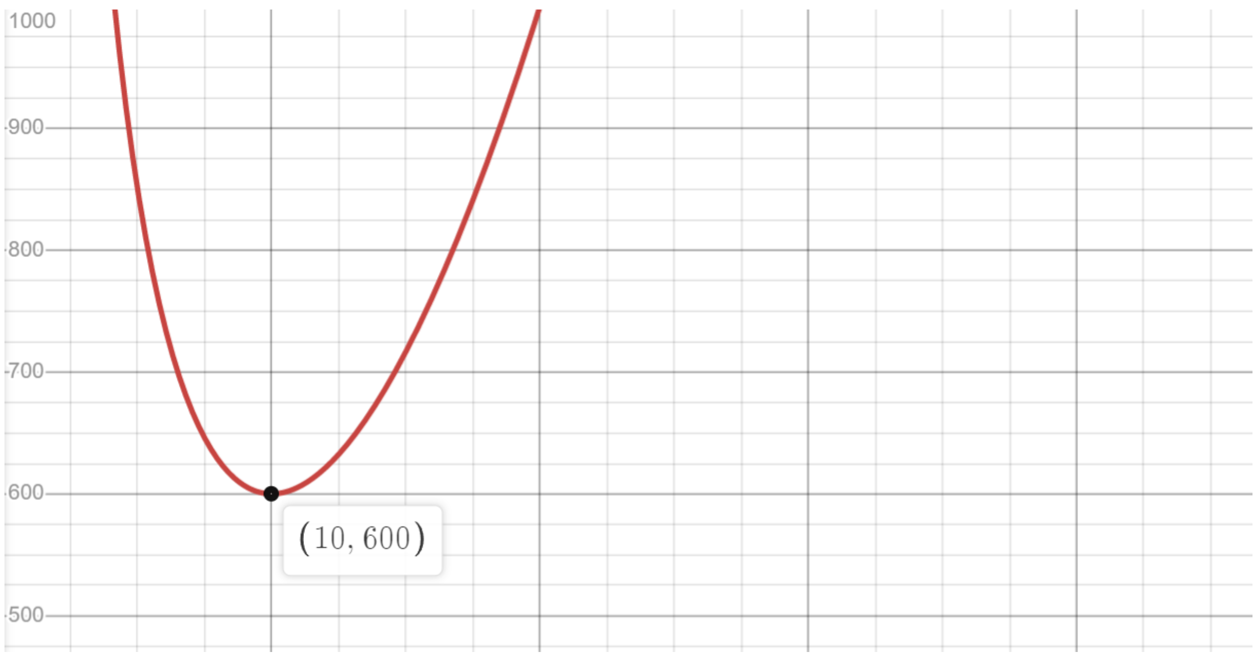


C. 5 min after injection.

D. C(5) = 5 mg/L

4. A. S(x) = $2x^2 + 4xy = 2x^2 + \frac{4000}{x}$

B. Graph



C. The least surface area results when x ≈ 10. Based on this number, S(10) = 600 in^2

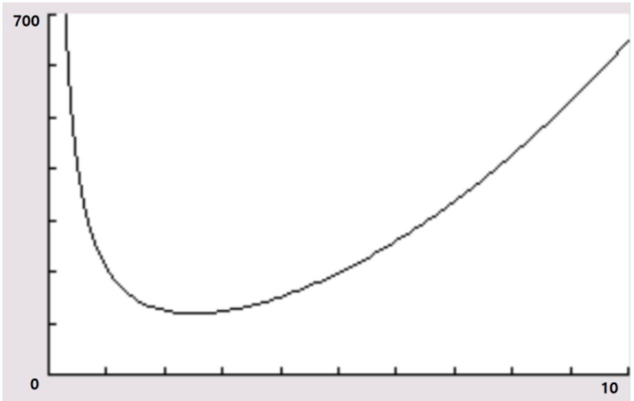
D. C(x) = $.64x^2 + \frac{1000}{x}$

Min cost results when x ≈ 9.2. *The minimum cost would be* C(9.2) ≈ $162.87.

5. A. A(r) = $2\pi r^2 + \frac{200}{r}$

B. 123.22 ft^2

C. Graph.



A is smallest when r ≈ 2.5 ft

CHAPTER PROJECT VIII

```
from fractions import Fraction
from sympy import *
print('Input (a*x+d)/((x-b)*(x-c))')
a = int(input('a? '))
d = int(input('d? '))
b = int(input('b? '))
c = int(input('c? '))
A = Fraction(d+b*c,b-c)
B = Fraction(a*c+d,c-b)
frac = '('+str(a)+'*x+'+str(d)+')/((x-'+str(b)+')*(x-'+str(c)+'))'
frac = sympify(frac)
pprint(frac)
pfd = str(A)+'/(x-'+str(b)+') + '+ str(B)+'/(x-'+str(c)+')'
pfd = sympify(pfd)
pprint(pfd)
```